

Relativistic time dilatation and the spectrum of electrons emitted by 33 TeV lead ions penetrating thin foils

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Abstract

We study the energy distribution of ultrarelativistic electrons produced when a beam of 33 TeV $\text{Pb}^{81+}(1s)$ ions penetrates a thin Al foil. We show that, because of a prominent role of the excitations of the ions inside the foil which becomes possible due to the relativistic time dilatation, the width of this distribution can be much narrower compared to the case when the ions interact with rarefied gaseous targets. We also show that a very similar shape of the energy distribution may arise when 33 TeV Pb^{82+} ions penetrate a thin Au foil. These results shed some light on the origin of the very narrow electron energy distributions observed experimentally about a decade ago.

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Atomic physics normally does not deal with objects exposed to extreme conditions. One of the interesting and important exceptions of this rule is represented by the studies of various phenomena accompanying the penetration of targets by highly charged projectile ions moving with velocities very close to the speed of light. During the interaction between the ion and a target atom both of these particles are exposed to extremely intense and extraordinarily short pulses of the electromagnetic fields.

For instance, in collisions of 33 TeV hydrogen-like $\text{Pb}^{81+}(1s)$ ions with Al (which will be considered below) the typical durations of the electromagnetic pulses acting on the electron bound in the ion are $\lesssim 10^{-21}$ sec (in the rest frame of the ion). The peak pulse intensities in this frame can reach $\sim 10^{28}\text{-}10^{29}$ W/cm² which enables, despite the very short interaction time, to induce transitions of the very tightly bound electron of the ion with a noticeable probability [1].

First experimental results on the total cross section for the electron loss from 33 TeV $\text{Pb}^{81+}(1s)$ were reported in [2] together with data for the electron capture by 33 TeV bare Pb^{82+} ions [3].

Compared to the study of the total cross sections much more information can be obtained when differential cross sections are explored. The first experimental results on the differential cross sections for such collisions were reported in [4]. In that experiment the incident beams of 33 TeV $\text{Pb}^{81+}(1s)$ and 33 TeV Pb^{82+} were penetrating Al and Au foils, respectively. In both cases it was found that the penetration is accompanied by the emission of ultrarelativistic electrons whose energy distributions have the form of a cusp with a maximum at an energy corresponding to the electrons moving in the laboratory frame with velocities equal to that of the ions.

One of unexpected results reported in [4] was that the measured distribution of the high-energy electrons produced under the bombardment of a thin Al foil was found to be much narrower than one could expect based on the consideration of the width of the Compton profile of the electron state in the incident $\text{Pb}^{81+}(1s)$ ions [4]. Moreover, in a more rigorous calculation performed in [5] for

the energy spectrum of electrons emitted from a 33 TeV $\text{Pb}^{81+}(1s)$ ion colliding with an Al atom it was confirmed that such a spectrum is indeed much broader than that observed in the experiment [4].

Another intriguing finding of [4] was that for 33 TeV Pb^{82+} ions incident on a thin Au foil the shape of the measured energy distributions of high-energy electrons emerged from the foil was **very similar** to that obtained for the beam of 33 TeV $\text{Pb}^{81+}(1s)$ ions incident on the Al foil.

It is known that the total and differential loss cross sections depend on a bound state from which the electron leaves the ion (see e.g. [6], [7] and references therein). Therefore, it was speculated in [4] that in the case of the incident 33 TeV Pb^{82+} ions the very narrow shape of the electron cusp might be a signature of the electron capture into excited states. However, for the $\text{Pb}^{81+}(1s)$ ions incident on the Al foil the possible influence of excited states of these ions on the electron cusp was not considered seriously because of the common experience that excitations of very heavy hydrogen-like ions inside thin foils of relatively light elements do not have a noticeable impact on the electron loss process.

For instance, in the recent experimental-theoretical study [8] on 200 MeV/u $\text{Ni}^{27+}(1s)$ ions incident on gaseous and solid targets it was found that the fraction of the ions excited inside the solids does not exceed 5-6%. Moreover, even such rather modest values seem to be hardly reachable for very heavy hydrogen-like ions since, compared to the case of relatively light ions, the penetration of matter by the very heavy ions possesses the following two important differences.

First, because of a very tight binding of the electron in such ions cross sections for collision-induced electron transitions are much smaller. Therefore, for highly charged ions, like Pb^{81+} , moving inside solids the mean free path with respect to the collision-induced transitions will be much larger. Second, the lifetimes of the excited states with respect to the spontaneous radiative decay in such ions are much shorter.

The above two points mean that there will be much

more time between the consequent collisions for the excited ion to relax into the ground state via the spontaneous radiative decay. As a result, there might seem to be sound grounds for the sceptic attitude to the possible role played by the excitations of the incident 33 TeV Pb⁸¹(1s) ions in the formation of the electron cusp. However, it will be demonstrated below that the expectations based on the experience accumulated when exploring collisions at moderate relativistic impact energies have to be substantially corrected in the case of the extreme relativistic energies studied in [4].

Our consideration of the energy spectrum of the cusp electrons assumes that the foil materials are amorphous (not crystals) and includes three main ingredients.

First, the basis of our consideration is represented by calculations of cross sections for the projectile-electron excitation (de-excitation) and loss occurring in the ion-atom collisions. Besides, we calculate also cross sections for the bound-free pair production (at the extreme relativistic collision energy which we consider the radiative and kinematic capture channels can be ignored). In these our calculations we use the Dirac-Coulomb wave functions to describe bound and continuum states of the electron (and the positron) in the field of the bare lead nucleus. We shall not elaborate further on the details of the cross section calculations and just refer to [9] for the description of the calculation methods. In addition, within our basic atomic physics analysis we also calculate rates for the spontaneous radiative decay of excited hydrogen-like lead ions to all possible internal states with lower energies.

In the second step we solve the kinetic equations which describe the population of the internal states of the ion inside the foil given as a function of time t or of the ionic coordinate $z = vt$ inside the foil (z and t are measured in the laboratory frame, $\mathbf{v} = (0, 0, v)$ is the projectile velocity). These equations read

$$\begin{aligned} \frac{dN_0}{dt} &= -\frac{N_0}{\tau_{capt}} + \sum_{j=1}^{N_{max}} \frac{N_j}{\tau_j^{loss}}, \\ \frac{dN_j}{dt} &= \frac{N_0}{\tau_j^{capt}} - \frac{N_j}{\tau_j^{loss}} - N_j \sum_{i=1}^{i \leq j} \frac{1}{\tau_{j \rightarrow i}^{sp}} \\ &\quad - N_j \sum_{i=1(i \neq j)}^{N_{max}} \frac{1}{\tau_{j \rightarrow i}} + \sum_{i=1(i \neq j)}^{N_{max}} \frac{N_i}{\tau_{i \rightarrow j}}. \end{aligned} \quad (1)$$

Here, N_0 is the number of bare ions, N_j is the number of ions with one electron in the j -th internal state ($j = 1, 2, \dots, N_{max}$) and N_{max} is the total number of the involved bound states. Further, τ_j^{capt} is the mean time for the electron vacuum capture into the j -th state, τ_j^{capt} is the mean time for the electron capture to any state ($1/\tau_j^{capt} = 1/\tau_1^{capt} + 1/\tau_2^{capt} + \dots$), τ_j^{loss} is the mean time for the electron loss from a state j to the continuum, $\tau_{j \rightarrow i}$ is the mean time for the collision induced transition from the internal state i to the internal state j and $\tau_{j \rightarrow i}^{sp}$ is the

lifetime of the state j with respect to the spontaneous radiative transition to any possible state i .

The elementary cross sections and spontaneous decay rates obtained during the first step of the consideration enable one to get the above mean excitation/de-excitation loss and capture times in the usual way. For instance, $\tau_j^{loss} = 1/(n_a \sigma_j^{loss} v)$, where σ_j^{loss} is the cross section for the electron loss from the j -th internal state of the ion and n_a is the atomic density of the target.

Once the functions $N_j(z)$ are known, the (preliminary) estimate for the energy spectrum of the electrons emitted from the ion traversing a solid foil of a thickness L is given by

$$\frac{dn_e}{d\varepsilon_p} = n_a \sum_{j=1}^{N_{max}} \frac{d\sigma_j^{loss}}{d\varepsilon_p} \int_0^L dz N_j(z), \quad (2)$$

where ε_p is the total electron energy in the laboratory frame and $\frac{d\sigma_j^{loss}}{d\varepsilon_p}$ is the energy distribution of the electrons emitted from the internal state j .

The third step of our consideration deals with the transport of the emitted electrons through the foil. The detailed analysis of this step represents in general quite a delicate task but in our case is substantially simplified by the fact that the electrons leaving the ions have in the laboratory frame extremely high values of energy. There are two main effects which can influence the shape of the electron energy distribution when the electrons penetrate the foil.

The first concerns energy losses of the ultrarelativistic electrons traversing the foil. These losses are caused by (i) the excitation of the electrons of the foil and (ii) the emission of the radiation by the ultrarelativistic electrons because of their acceleration during the interactions with the atomic nuclei in the foil. However, for the foil parameters used in the experiment [4] the energy losses can simply be ignored because they are very small ($\lesssim 0.5\%$) compared to the initial energies of these electrons.

The second effect which may possibly influence the shape of the measured energy distributions is that collisions in the foil broaden the distribution of the ultrarelativistic electrons over the transverse components (p_x, p_y) of their momenta. For the foil parameters used in [4], the multiple collisions suffered by the ultrarelativistic electrons inside the foil substantially increase the width of their (p_x, p_y)-distribution compared to that which these electrons have when leaving the 33 TeV nuclei.

Nevertheless, even after this increase the transverse components ($\sim 10^2$ a.u.) remain very small compared to the total electron momenta ($\simeq 2 \times 10^4$ a.u.) which means that the broadening of the (p_x, p_y)-distribution may have an impact on the measured electron momentum distribution only if special geometric conditions are employed in an experiment [10]. Since we do not possess all necessary information about the real conditions of the experiment [4], in our calculations for the energy spectra,

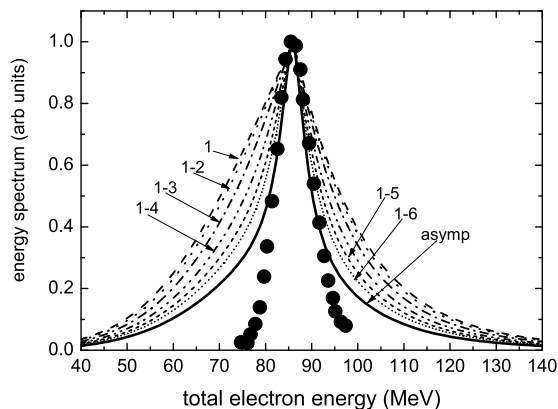


FIG. 1: The energy distribution of the electron cusp produced in collisions between an incident beam of 33 TeV $\text{Pb}^{81+}(1s)$ with Al foil with a thickness of 2.85×10^{-2} cm (for more explanation see the text). Circles show the electron energy distribution measured in [4] for 33 TeV Pb^{81+} colliding with the Al foil of the same thickness. All the distributions are given in the laboratory frame and are normalized to 1 at the maximum.

discussed below, we simply take all electrons (whichever angle they have after leaving the foil) into account.

Under such conditions the changes in the electron momenta during the electron transport through the foil do not have an impact on the final electron energy distribution. Therefore, the main essential difference between the previous estimates for the shape of the electron cusp in the case of 33 $\text{Pb}^{81+}(1s)$ incident on an Al foil (where it was assumed that the electron loss occurs in the single collision regime) and our present model is that the latter takes into account electron transitions to the continuum not only directly from the ground state of the ions but also via the intermediate excitations to higher bound states occurring when the ions penetrate the foil.

The initial expectations, that in the case of very heavy ions their excitations are of minor importance for the loss process, seems to be just confirmed if we compare in figure 1 curves labeled with '1' and '1-2'. In this figure, where we present results for the electron energy spectrum in the case of 33 $\text{Pb}^{81+}(1s)$ ions incident on Al foil, the curve '1' was obtained by ignoring all excited bound states while in the calculation resulted in the curve '1-2' the states with the principal quantum number $n = 2$ were also taken into account. Yet, there is just a tiny difference in the widths of these two curves.

However, when we add the states with $n = 3$ into our analysis (the curve in figure 1 labeled by '1-3') the width-reducing effect becomes quite visible. Adding into the analysis the states with $n = 4$ leads to a further reduction in the calculated width and this reduction is even larger than that observed when the states with $n = 3$ were added. The reduction of the width continues further when we add states with $n = 5$ and $n = 6$ (see figure 1), however, it proceeds at a smaller pace compared to

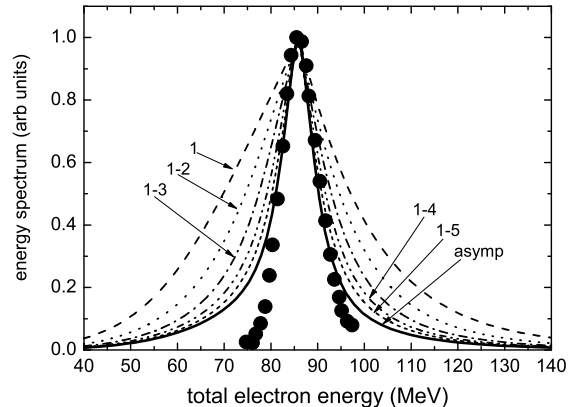


FIG. 2: Same as in figure 1 but for an incident beam of 33 TeV Pb^{82+} penetrating Au foil with a thickness of 8.81×10^{-4} cm corresponding to the conditions of the capture experiment [4]. For more information, circles show the electron spectrum measured in [4] for 33 TeV $\text{Pb}^{81+}(1s)$ ions incident on the Al foil.

that when the states with $n = 3$ and $n = 4$ were added.

Note that the inclusion of the states with $n = 1-6$ into the analysis means that we calculated the collision-induced and spontaneous radiative transitions in the system of levels involving 182 quantum bound states of the Pb^{81+} ion as well as the electron and positron continua in the field of the nucleus Pb^{82+} . This is quite demanding and computationally expensive task. Due to obvious reasons in our calculations we cannot increase indefinitely the number of bound states. Therefore, we have applied an extrapolation procedure in order to get the asymptotic limit for the electron cusp shape effectively corresponding to taking into account all bound states ($n = 1-\infty$). The result of this extrapolation is shown in figure 1 by the curve labeled 'asympt'.

Comparing the energy distributions in figure 1 we see that their asymptotic width is about a factor of 3 smaller than the width obtained by assuming that the cusp is produced under the single-collision conditions. This strong effect is caused by the excitation of the ions inside the foil which involves rather highly lying bound states: when the ions move in the foil the electron cloud surrounding the ionic nuclei has enough time to expand tremendously in size before it will almost completely disappear due to the transitions to the continuum. The key factor making this possible is the relativistic time dilatation which effectively decreases the spontaneous decay rates of the excited states of the ions by a factor of ≈ 170 .

One more point which should be mentioned is that cross sections for the electron capture are relatively very small. As a result, in the formation of the electron cusp in the case of the hydrogen-like ions incident on the Al foil the capture channels do not play any noticeable role.

In figure 2 we show results for the energy spectrum calculated for 33 TeV Pb^{82+} incident on a Au foil. Of course, now the electron capture from vacuum becomes

of paramount importance for the very existence of the electron cusp. One should note, however, that the capture cross sections decrease very rapidly when n and j_e increase (j_e is the total angular momentum of the electron in a bound state). Therefore, the most of the excited bound states having a very important impact on the energy spectrum are populated not by capturing the electron directly from the vacuum but via the excitations from few states with the lowest values of n and j_e for which the capture is efficient. This indirect way becomes especially effective because in collisions with Au atoms the excitation cross sections are much larger than in the case with Al.

Comparing the spectra shown in figure 2 with those displayed in figure 1 we see that the changes in the form of the calculated spectrum in figure 2 (occurring when we allow for more bound states in our analysis) are accumulating at a different pace. Besides, the asymptotic cusp shape in figure 2 has less pronounced wings. These differences are related to two basic reasons: (i) the excitation/loss cross sections in an Au foil are much larger while the spontaneous decay rates remain exactly the same as in the case of an Al foil and (ii) the initial step in the cusp formation is now represented by the capture process which also somewhat increases the relative population of the excited states compared to the case when the beam of $\text{Pb}^{81+}(1s)$ ions was incident on the Al foil.

Curiously, however, that the asymptotic width in figure 2 is again about 3 times smaller than the 'initial' width and the shape of the asymptotic spectra in both cases looks similar (which is also in agreement with the experimental observations of [4]). In general such a similarity will not hold when the foil parameters (for instance, their

thicknesses) are changed and, in this sense, is accidental. Yet, in both cases the strong reductions in the widths of the energy distributions are caused by the excitations of the electrons to rather high lying bound ionic states occurring when the ions penetrate the foils.

In summary, briefly, we have considered the energy spectra of the ultrarelativistic electrons emitted when incident 33 TeV $\text{Pb}^{81+}(1s)$ and Pb^{82+} ions penetrate Al and Au foils, respectively. The foil thicknesses were taken to be the same as used in the experiment [4]. We have found that these spectra are much narrower than those which would be produced under the single-collision conditions and have similar shapes. The similarity in the shapes in general will not hold if foils with other parameters would be used and, thus, is fortuitous. However, in both cases the strong width reduction is caused by the excitations of the ions when they penetrate the target foils suffering multiple collisions with the target atoms. Such a profound role of the excitations in the case of very heavy ions is in contrast to the previous experience gained when exploring collisions in the low and intermediate relativistic domains of impact energies. In the case under consideration the excitations become so effective because of the relativistic time dilatation which decreases very strongly the spontaneous decay rates of excited states in the ions moving with velocities closely approaching the speed of light.

Our present results shed some light on the origin of the unexpectedly narrow shape of the electron cusp produced by the ultrarelativistic heavy ions. However, a more careful analysis taking into account all real conditions of the experiment [4] would be necessary in order to make a detailed comparison between the experiment and theory.

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- [1] These field parameters may be compared with the parameters of state of the art laser systems whose shortest pulse lengths are about $\sim 10^{-16}$ - 10^{-15} sec and whose peak intensities do not exceed $\sim 10^{21}$ - 10^{22} W/cm².
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 - [10] For instance, if an experiment detects only those high-energy electrons, which finally move inside a very narrow cone centered along \mathbf{v} , the expansion the electron (p_x, p_y)-distribution inside the foil will first of all impact the detected numbers of the electrons emitted from the ground state of the lead ions (both in absolute and relative proportions) because these electrons right after the emission from the ions have larger transverse momenta (and, besides, are the first to appear in the continuum). Compared to the emission from excited states the electrons ejected from the ground state have the largest width of the energy distribution. Therefore, a comparatively stronger removal of those electrons, which were emitted from the ground state, occurring when the electron beam traverses the foil will effectively decrease the width of the measured electron energy distribution.